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From (4), $z = \frac{ax(2x-c)}{2x(2x-c)-bd}$, which substituted in (5), gives, after reduction,

$$x^4 + \frac{1}{2f}(ae-2cf)x^3 + \frac{1}{4f}[fc^2 - a^2d - 2(ace + bdf)]x^2 \\ + \frac{1}{8f}[ac(ad+ce) + bd(2cf-ae)]x + \frac{1}{16f}bd(ace + bdf) = 0.$$

Similarly,

$$y^4 + \frac{1}{2b}(ac-2be)y^3 + \frac{1}{4b}[fc^2 - e^2b - 2(ace + bdf)]y^2 \\ + \frac{1}{8b}[ce(ae+cf) + df(2be-ac)]y + \frac{1}{16b}df(ace + bdf) = 0.$$

$$z^4 + \frac{1}{2d}(ce-2ad)z^3 + \frac{1}{4d}[a^2d - e^2b - 2(ace + bdf)]z^2 \\ + \frac{1}{8d}[ac(ae+be) + bf(2ad-ce)]z + \frac{1}{16d}bf(ace + bdf) = 0.$$

Also solved by LON C. WALKER.

GEOMETRY.

REMARKS ON NO. 187, GEOMETRY, BY J. R. HITT, GOSS, MISS.

There seems to be an error in (4) of Professor Zerr's demonstration of No. 187, Geometry. The result given is not correct. For $t = \frac{1}{\sqrt{2}}b\sin C\cos C$, $t_1 = \frac{1}{\sqrt{2}}b\cos C$, $t_2 = \frac{1}{\sqrt{2}}b\sin C$, from which it is seen that in general t^2 cannot equal t_1t_2 . It is also easily seen that if $t_1 : t = t : t_2$, then must $DI = b$, whereas DI cannot be $> \frac{1}{2}b$.

CALCULUS.

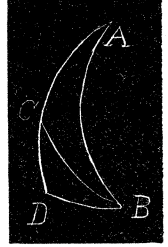
154. Proposed by B. R. DOWNER, Hopkinsville, Ky.

At the equinox, when the sun is on the celestial equator, a man starts driving on a perfectly level plain at six o'clock in the morning, and continues, going always from the sun, at the uniform rate of six miles per hour, until six o'clock in the evening. Required the path he will travel and the distance in a straight line from starting point to stopping point.

Solution by the PROPOSER.

Let the line from west to east through the starting point be taken for the axis of X , and that from south to north through the same point for the axis of Y . Let δ be the latitude of the place; then will the inclination of the celestial equator to the vertical circle passing through the east and west points be equal to δ .

Let AB be an arc of this vertical circle extending from the zenith A to the east point B , 90° . Let BC be an arc of the celestial equator, inclined at the angle δ to AB , and AC the vertical circle passing through the sun C at any particular time. If then, z represents the arc of the curve traveled by the man in any given time, BC , the sun's apparent path, expressed in terms of z , will be, for the radius 1, $\pi z/72$. Produce AC until AD is 90° , and draw BD . Then will the angles ABD and D be right angles and the arc BD will measure the angle A , the sun's angular distance at any time from the east point. Then we have from the right angled spherical triangle BCD ,



$$\tan A = \sin \delta \tan \frac{\pi z}{72}.$$

But evidently the angle A is the supplement of that which the tangent at any point of the required curve makes with the axis of X .

$$\therefore \frac{dy}{dx} = -\sin \delta \tan \frac{\pi z}{72}. \quad \text{Whence } z = \frac{72}{\pi} \tan^{-1} \left(-\frac{dy/dx}{\sin \delta} \right).$$

$$\therefore dz = -\frac{72}{\pi} \frac{\sin \delta \, d^2 y}{dx(\sin^2 \delta + (dy^2/dx^2))}. \quad \text{But } dz = -\sqrt{(dx^2 + dy^2)}.*$$

$$\therefore \sqrt{1 + \frac{dy^2}{dx^2}} = \frac{72}{\pi} \frac{\sin \delta \, \frac{d^2 y}{dx^2}}{\sin^2 \delta + \frac{dy^2}{dx^2}}.$$

$$\text{Let } \frac{dy}{dx} = p. \quad \text{Then } \sqrt{1+p^2} = \frac{72}{\pi} \frac{\sin \delta (dp/dx)}{\sin^2 \delta + p^2}.$$

$$\therefore dx = \frac{72}{\pi} \sin \delta \frac{dp}{(\sin^2 \delta + p^2)\sqrt{1+p^2}}$$

$$\text{and } p dx = dy = \frac{72}{\pi} \sin \delta \frac{p dp}{(\sin^2 \delta + p^2)\sqrt{1+p^2}}.$$

* The negative sign of the radical is taken because of the reverse direction in which we have measured the arc z .

Hence we have $x = \frac{72}{\pi} \sin \delta \int \frac{dp}{(\sin^2 \delta + p^2) \sqrt{1+p^2}}$.

Put $\sqrt{1+p^2} = r - p$. Then $p = \frac{r^2 - 1}{2r}$, $dp = \frac{r^2 + 1}{2r^2} dr$, $\sqrt{1+p^2} = \frac{r^2 + 1}{2r}$,

$$\sin^2 \delta + p^2 = \frac{r^4 - 2r^2 \cos 2\delta + 1}{4r^2}.$$

Whence $\int \frac{dp}{(\sin^2 \delta + p^2) \sqrt{1+p^2}} = \int \frac{4rdr}{r^4 - 2r^2 \cos 2\delta + 1} = \int \frac{4rdr}{(r^2 \cos 2\delta)^2 + \sin^2 2\delta}$

Put $r^2 - \cos 2\delta = s$. Then $2rdr = ds$, and the integral becomes

$$\int \frac{2ds}{s^2 + \sin^2 2\delta} = \frac{2}{\sin 2\delta} \int \frac{ds/\sin 2\delta}{1 + [s/(\sin 2\delta)]^2} = \frac{2}{\sin 2\delta} \tan^{-1} \frac{s}{\sin 2\delta} + c.$$

$$\therefore x = \frac{72}{\pi \cos \delta} \tan^{-1} \left[\frac{p^2 + p\sqrt{1+p^2} + \sin^2 \delta}{\sin \delta \cos \delta} \right] + c.$$

But from the conditions of the problem, when $x=0$, $p=0$.

$$\therefore c = -\frac{72\delta}{\pi \cos \delta}, \text{ and } x = \frac{72}{\pi \cos \delta} \tan^{-1} \left[\frac{p^2 + p\sqrt{1+p^2} + \sin^2 \delta}{\sin \delta \cos \delta} \right] - \frac{72\delta}{\pi \cos \delta}.$$

Again we have $y = \frac{72}{\pi} \sin \delta \int \frac{pdp}{(\sin^2 \delta + p^2) \sqrt{1+p^2}}$.

Put $\sqrt{1+p^2} = r$. Then $pdp = rdr$. $\sin^2 \delta + p^2 = r^2 - \cos^2 \delta$.

$$\begin{aligned} \int \frac{pdp}{(\sin^2 \delta + p^2) \sqrt{1+p^2}} &= \int \frac{dr}{r^2 - \cos^2 \delta} = \frac{1}{2 \cos \delta} \int \frac{dr}{r - \cos \delta} - \frac{1}{2 \cos \delta} \int \frac{dr}{r + \cos \delta} \\ &= \frac{1}{2 \cos \delta} \log \left[\frac{r - \cos \delta}{r + \cos \delta} \right] + c'. \end{aligned}$$

$$\therefore y = \frac{36 \tan \delta}{\pi} \log \left[\frac{\sqrt{1+p^2} - \cos \delta}{\sqrt{1+p^2} + \cos \delta} \right] + c'.$$

But when $y=0$, $p=0$.

$$\therefore c' = -\frac{36 \tan \delta}{\pi} \log \tan^2 \frac{1}{2} \delta.$$

$$\therefore y = \frac{36 \tan \delta}{\pi} \log \left[\frac{\sqrt{1+p^2} - \cos \delta}{\sqrt{1+p^2} + \cos \delta} \right] - \frac{36 \tan \delta}{\pi} \log \tan^2 \frac{1}{2} \delta.$$

From the two equations for x and y , we have

$$p^2 + p \sqrt{1+p^2} = \sin \delta \cos \delta \tan \left[\frac{\pi x \cos \delta}{72} + \delta \right] - \sin^2 \delta \dots (i).$$

$$\sqrt{1+p^2} = \cos \delta \frac{\cot^2 \frac{1}{2} \delta + e^{\pi y / 36 \tan \delta}}{\cot^2 \frac{1}{2} \delta - e^{\pi y / 36 \tan \delta}} \dots (ii).$$

Denote the second members of equations (i) and (ii) by X and Y . Then we have $p^2 + p \sqrt{1+p^2} = X$, $\sqrt{1+p^2} = Y$. Whence, by eliminating p , we have

$$Y^2 = \frac{(X+1)^2}{2X+1} \dots (iii).$$

which is an equation expressing the general character of the curve, which is transcendental. Making $x=0$, we may find two values of y corresponding, which are evidently the starting and stopping points in the problem proposed.

When $x=0$, $X=0$. Hence from (iii) we find $Y = \pm 1$, or

$$\cos \delta \frac{\cot^2 \frac{1}{2} \delta + e^{\pi y / 36 \tan \delta}}{\cot^2 \frac{1}{2} \delta - e^{\pi y / 36 \tan \delta}} = \pm 1;$$

whence $e^{\pi y / 36 \tan \delta} = 1$ or $\cot^4 \frac{1}{2} \delta$.

$\therefore y=0$, or $\frac{144 \tan \delta}{\pi} \log \cot^2 \frac{1}{2} \delta$, and the distance from starting point to stopping point is the latter value of y .

For the latitude $36^\circ 20'$ we have for the distance from starting point to stopping point 37.56 miles.

MECHANICS.

144. Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Pressure is applied perpendicularly to the plane surface yz , bounding an otherwise infinite isotropic solid. Find the resultant displacements, if the pressure varies as $\sin\left(\frac{2\pi y}{a}\right) + \sinh\left(\frac{2\pi y}{a}\right)$.

Solution by the PROPOSER.

Measuring the axis of x perpendicularly to the plane face into the interior of the solid, the longitudinal stresses P , Q and the shearing stress T are functions of x and y only, and the general equations of internal equilibrium reduce to (Thompson and Tait, *Natural Philosophy*, Art. 697) the following: